

Code No: 182AR

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year II Semester Examinations, September - 2023

ORDINARY DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS

(Common to EEE, CSE, IT, CSIT, CE (SE), CSE (CS), CSE (DS), CSD)

Time: 3 Hours

Max. Marks: 60

**Note:** This question paper contains two parts A and B.i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

**PART- A****(10 Marks)**

- 1.a) Assume that all the zeroes of the polynomial  $a_0m^2 + a_1m + a_2$  have negative real parts. If  $y(x)$  is a solution to the ordinary differential equation  $a_0D^2y + a_1Dy + a_2y = 0$ , then find the value of  $\lim_{x \rightarrow \infty} (y(x) - 1)$ . [1]
- b) If the integrating factor of  $(x^7y^2 + 3y)dx + (3x^8y - x)dy = 0$  is  $x^{m+1}y^{n+1}$ , find the values of  $m$  and  $n$ . [1]
- c) Find the general solution of  $y'' + 4y' + 13y = 0$ . [1]
- d) Find the general solution of  $y'' + 4y = \cos x$ . [1]
- e) Find the Laplace transform of  $f(t) = \sinh^2 t$ . [1]
- f) Find the Laplace transform of  $f(t) = \cos at, t \geq 0$ . [1]
- g) Find the gradient of the scalar function  $f(x, y, z) = \ln(x^2 + y^2 + z^2)$  at the point  $(3, -4, 5)$ . [1]
- h) Find the divergence of the vector field  $\mathbf{v} = xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k}$ . [1]
- i) State Green's Theorem. [1]
- j) Evaluate the line integral  $\int_C \mathbf{v} \cdot d\mathbf{r}$ , where  $\mathbf{v} = 2x^2y\mathbf{i} - xy^2\mathbf{j}$  and  $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}$ ,  $0 \leq t \leq 4$ . [1]

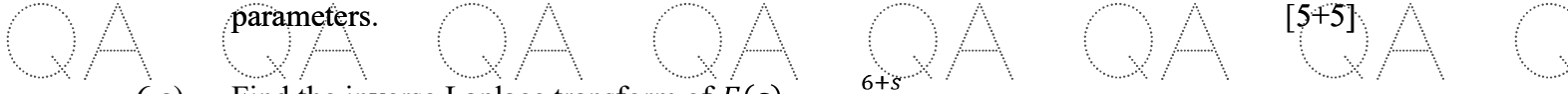
**PART - B****(50 Marks)**

- 2.a) Show that the differential equation  $xdy - ydx = 0$  is not exact. Find three different integrating factors which are not constant multiple of each other.
- b) Solve the differential equation  $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$ . [5+5]
- OR**
- 3.a) Find the orthogonal trajectories of the family of curves  $e^{2x} \sin 2y = c$ .
- b) Uranium disintegrates at a rate proportional to the amount then present at any instant. If  $M_1$  and  $M_2$  grams of uranium are present at times  $T_1$  and  $T_2$  respectively, find the half-life of uranium. [5+5]
- 4.a) Find the general solution of  $2y'' + 3y' - 2y = e^x + e^{-2x}$ .
- b) Find the general solution of  $x^2y'' + 5xy' + 3y = \ln x, x > 0$ . [5+5]



OR

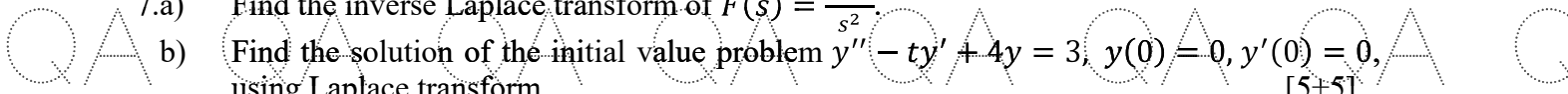
- 5.a) Find the general solution of  $y'' + 3y' + 2y = x e^x \sin x$ .  
b) Find the general solution of  $y'' + 4y' + 4y = e^{-2x} \sin x$ , using the method of variation of parameters. [5+5]



- 6.a) Find the inverse Laplace transform of  $F(s) = \frac{6+s}{s^2+6s+13}$ .  
b) Find the solution of the initial value problem  $y'' + 4y' + 13y = e^{-t}$ ,  $y(0) = 0$ ,  $y'(0) = 2$ , using Laplace transform. [5+5]

OR

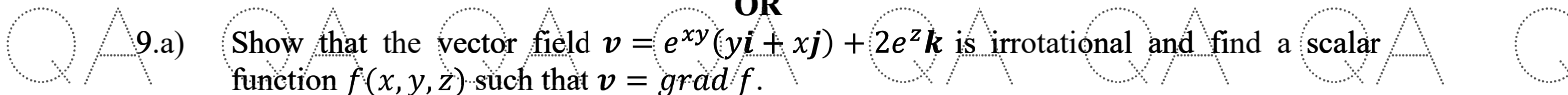
- 7.a) Find the inverse Laplace transform of  $F(s) = \frac{e^{-2s}}{s^2}$ .  
b) Find the solution of the initial value problem  $y''' - ty' + 4y = 3$ ,  $y(0) = 0$ ,  $y'(0) = 0$ , using Laplace transform. [5+5]



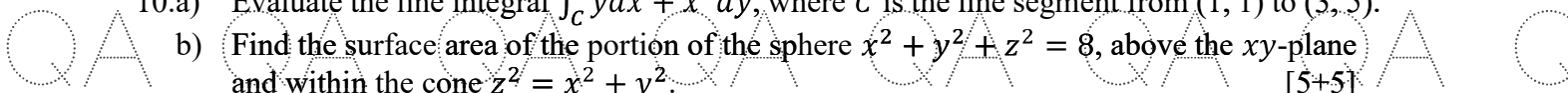
- 8.a) Prove the vector identity  $\nabla \cdot [(f\nabla g) \times (g\nabla f)] = 0$ .  
b) Find the directional derivative of the function  $f(x, y, z) = x^2 + y^2 + 2z^2$  at the point  $(1, 1, 2)$  in the direction of  $\text{grad } f$ . [5+5]

OR

- 9.a) Show that the vector field  $\mathbf{v} = e^{xy}(y\mathbf{i} + x\mathbf{j}) + 2e^z\mathbf{k}$  is irrotational and find a scalar function  $f(x, y, z)$  such that  $\mathbf{v} = \text{grad } f$ .  
b) Find the values of the constants  $a, b$  and  $c$  such that the maximum value of the directional derivative of  $f(x, y, z) = axy^2 + byz + cx^2z^2$  at  $(1, -1, 1)$  is in the direction parallel to the axis of  $y$  and has magnitude 6. [5+5]



- 10.a) Evaluate the line integral  $\int_C y dx + x^2 dy$ , where  $C$  is the line segment from  $(1, 1)$  to  $(3, 5)$ .  
b) Find the surface area of the portion of the sphere  $x^2 + y^2 + z^2 = 8$ , above the  $xy$ -plane and within the cone  $z^2 = x^2 + y^2$ . [5+5]



OR

11. Use Stoke's theorem to evaluate  $\int_C [(x+y)dx + (2x-z)dy + (y+z)dz]$ , where  $C$  is the boundary of the triangle with vertices  $(2, 0, 0)$ ,  $(0, 3, 0)$ ,  $(0, 0, 6)$ . [10]

