R22

Code No: 182AR

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech I Year II Semester Examinations, September - 2023

ORDINARY DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS

(Common to EEE, CSE, IT, CSIT, CE (SE), CSE (CS), CSE (DS), CSD)

Time: 3 Hours **Max. Marks: 60**

Note: This question paper contains two parts A and B.
i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART- A

(10 Marks)

1.a) Assume that all the zeroes of the polynomial $a_0m^2 + a_1m + a_2$ have negative real parts. If $y(x)$ is a solution to the ordinary differential equation $a_0D^2y + a_1Dy + a_2y = 0$, then find the value of $\lim_{x\to\infty}(y(x) - 1)$. [1]

b) If the integrating factor of $(x^7y^2 + 3y)dx + (3x^8y - x)dy = 0$ is $x^{m+1}y^{n+1}$, find the values of m and n. [1]

c) Find the general solution of $y'' + 4y' + 13y = 0$. [1]

d) Find the general solution of $y'' + 4y = \cos x$. [1]

e) Find the Laplace transform of $f(t) = \sinh^2 t$. [1]

f) Find the Laplace transform of $f(t) = \cos at\,, t \geq 0$. [1]

g) Find the gradient of the scalar function $f(x, y, z) = \ln(x^2 + y^2 + z^2)$ at the point $(3, -4, 5)$. [1]

h) Find the divergence of the vector field $\boldsymbol{v} = xy\boldsymbol{i} + yz\boldsymbol{j} + zx\boldsymbol{k}$. [1]

i) State Green's Theorem. [1]

j) Evaluate the line integral $\int_C \boldsymbol{v}\cdot d\boldsymbol{r}$, where $\boldsymbol{v} = 2x^2y\boldsymbol{i} - xy^2\boldsymbol{j}$ and $\boldsymbol{r}(t) = t\,\boldsymbol{i} + t^2\,\boldsymbol{j}$, $0 \leq t \leq 4$. [1]

PART - B

(50 Marks)

2.a) Show that the differential equation $xdy - ydx = 0$ is not exact. Find three different integrating factors which are not constant multiple of each other.

b) Solve the differential equation $y(xy + 2x^2y^2)dx + x(xy - x^2y^2)dy = 0$. [5+5]

OR

3.a) Find the orthogonal trajectories of the family of curves $e^{2x}\sin 2y = c$.

b) Uranium disintegrates at a rate proportional to the amount then present at any instant. If M_1 and M_2 grams of uranium are present at times T_1 and T_2 respectively, find the half-life of uranium. [5+5]

4.a) Find the general solution of $2y'' + 3y' - 2y = e^x + e^{-2x}$.

b) Find the general solution of $x^2y'' + 5xy' + 3y = \ln x,\ x > 0$. [5+5]

OR

5.a) Find the general solution of $y'' + 3y' + 2y = x\, e^{x} \sin x$.

b) Find the general solution of $y'' + 4y' + 4y = e^{-2x} \sin x$, using the method of variation of parameters. [5+5]

6.a) Find the inverse Laplace transform of $F(s) = \frac{6+s}{s^2+6s+13}$.

b) Find the solution of the initial value problem $y'' + 4y' + 13y = e^{-t}$, $y(0) = 0$, $y'(0) = 2$, using Laplace transform. [5+5]

OR

7.a) Find the inverse Laplace transform of $F(s) = \frac{e^{-2s}}{s^2}$.

b) Find the solution of the initial value problem $y'' - ty' + 4y = 3$, $y(0) = 0$, $y'(0) = 0$, using Laplace transform. [5+5]

8.a) Prove the vector identity $\nabla \cdot [(f\nabla g) \times (g\nabla f)] = 0$.

b) Find the directional derivative of the function $f(x, y, z) = x^2 + y^2 + 2z^2$ at the point $(1,1,2)$ in the direction of grad f. [5+5]

OR

9.a) Show that the vector field $\boldsymbol{v} = e^{xy}(y\boldsymbol{i} + x\boldsymbol{j}) + 2e^{z}\boldsymbol{k}$ is irrotational and find a scalar function $f(x, y, z)$ such that $\boldsymbol{v} = grad\, f$.

b) Find the values of the constants a, b and c such that the maximum value of the directional derivative of $f(x, y, z) = axy^2 + byz + cx^2z^2$ at $(1, -1, 1)$ is in the direction parallel to the axis of y and has magnitude 6. [5+5]

10.a) Evaluate the line integral $\int_C y dx + x^2 dy$, where C is the line segment from (1, 1) to (3, 5).

b) Find the surface area of the portion of the sphere $x^2 + y^2 + z^2 = 8$, above the xy-plane and within the cone $z^2 = x^2 + y^2$. [5+5]

OR

11. Use Stoke's theorem to evaluate $\int_C [(x + y)dx + (2x - z)dy + (y + z)dz]$, where C is the boundary of the triangle with vertices $(2,0,0), (0,3,0), (0,0,6)$. [10]

---ooOoo---